Continuous Spectrum—Kirchoff’s First Law

• A *hot opaque* substance (solid, liquid or gas) gives off a *continuous spectrum*, that is, a spectrum containing energy at all wavelengths.

• Examples where a continuous spectrum will be found:
  • An incandescent bulb.
  • A pool of molten iron
  • Lava flowing from a volcano.
  • A hot, ionized, opaque gas (such as below the Sun’s atmosphere).

• A continuous spectrum approximates a *black body* spectrum, the spectrum observed from a perfectly black substance heated to a high temperature. No real spectrum is a perfect black body spectrum, but many are good approximations to it.
Black-body (Continuous) Spectrum

- The energy distribution of a black body spectrum looks like this:

![Graph showing black-body spectrum with UV, Visible, and Infrared regions.](image-url)
Wien’s Law—Temperature Dependence (1)

• We are familiar with the fact that the color of a heated object changes as its temperature changes. As we heat a substance, it first glows a dull red; then a bright red, then orange, yellow, white, and finally bluish-white. This happens because the distribution of energy with wavelength shifts to predominantly shorter and shorter wavelengths as the temperature rises.

• Mathematically, we can calculate the wavelength of maximum emission:

\[
\text{Wavelength of Maximum} = \frac{3 \times 10^7}{T} \text{ Å}
\]

where T is the temperature in degrees Kelvin.
How Color Varies with Temperature
Emission Spectra—Kirchoff’s Second Law

- When a transparent gas is heated to a high temperature, one sees an *emission spectrum*, also known as a *bright-line spectrum*. Examples are:
  - “Neon” advertising signs
  - Some types of outdoor lighting
  - Gaseous nebulae in astronomy
NGC 6369
Chemical Fingerprints

- From such an object we observe a bright line or emission spectrum; light is emitted only at specific discrete wavelengths.
- The pattern of wavelengths at which the bright lines appear is distinctive and different for every element and molecule. Therefore, we can use the emission spectrum to determine the composition of astronomical objects. Here are some examples (wavelengths in Å):

- Hydrogen: 4102, 4340, 4861, 6563 Å
- Helium: 4471, 4686 (He⁺), 5876, 6678 Å
- Mercury: 4358, 5461, 5790 Å
- Sodium: 5890 Å
Chemical Fingerprints

- Here for example is what happens when you look at a mercury-vapor street lamp through a diffraction grating
Absorption Spectra—Kirchoff’s Third Law (1)

- When light passes through a cool gas, some of the light will be absorbed. The interesting thing is that the light is absorbed at the very same wavelengths that it would be emitted at had the gas been hot and giving off an emission spectrum. Thus, the spectra above would have the following appearance:

```
Hydrogen
4102 4340 4861 6563

He
4471 4686 (He\textsuperscript{+}) 5876 6678

Mercury
4358 5461 5790

Sodium
5890
```
Spectrum of the Sun
Absorption Spectra—Kirchoff’s Third Law (2)

- The energy distribution curve has notches cut out where the dark lines appear:

![Graph showing energy distribution with notches at specific wavelengths.](image-url)
Absorption and Emission Spectra

- Absorption and emission spectra sometimes go hand in hand. The light that is *absorbed* by the cool gas will be reemitted in another direction, and someone looking at that light will see an emission spectrum:
The Spectrum of the Sun

- The spectrum of the Sun and stars is a dark-line spectrum.
- This means that there is a hot opaque source of an emission spectrum, overlain by a cooler atmosphere of transparent gas.
- We can detect many different elements in the Sun’s spectrum: Hydrogen (80%) and Helium (most of the remainder) are the most abundant (Helium was first discovered on the Sun); elements like sodium, magnesium, iron; oxygen, carbon, neon, silicon, etc. are all seen, but only exist at the level of 1-2%.
A Model of the Sun (1)

Here is a simple model of the Sun: A sphere of hot, ionized (and therefore opaque) gas surrounded by a cooler (but still very hot), not ionized (and therefore transparent) atmosphere. Thus we see an absorption spectrum. When the photosphere is covered up by the Moon during an eclipse, we see an emission spectrum called the \textit{flash spectrum}. 

![Diagram of the Sun's atmosphere and photosphere with absorption and emission spectra labeled.]

Atmosphere

Photosphere (5000 K)

Absorption spectrum seen

Emission (flash) spectrum seen
Absorption and Emission Spectra

• Flash spectrum of Sun
How Spectra Vary With Temperature
### Spectral Types (2)

<table>
<thead>
<tr>
<th>Type</th>
<th>Temp</th>
<th>Color</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>25,000-50,000 K</td>
<td>Blue</td>
<td>Lines of ionized helium; plus lines of multiply ionized elements, such as O++, N++, C++, Si++. Balmer lines of hydrogen weak.</td>
</tr>
<tr>
<td>B</td>
<td>11,000-25,000 K</td>
<td>Blue</td>
<td>Lines of neutral helium; plus lines of ionized elements, such as O+, N+, C+, Fe++. Balmer lines of hydrogen moderately strong.</td>
</tr>
<tr>
<td>A</td>
<td>7,500-11,000 K</td>
<td>Blue-White</td>
<td>Balmer lines very strong; other features very weak or absent.</td>
</tr>
<tr>
<td>F</td>
<td>6,000-7,500 K</td>
<td>White</td>
<td>Balmer lines moderately strong; plus lines of some ionized elements, such as Ca+, Ti+, Fe+. (Ca+ at 3993 Å is about as strong as Hg at 4340 Å.)</td>
</tr>
</tbody>
</table>
### Spectral Types (3)

<table>
<thead>
<tr>
<th>Type</th>
<th>Temp</th>
<th>Color</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>5,000-6,000 K</td>
<td>Yellow-White</td>
<td>Balmer lines present, but weaker than in hotter stars. Lines of neutral (un-ionized) elements strong (e.g., Fe, Ti, Mg). Lines of easily ionized elements (e.g., Ca+) are strong.</td>
</tr>
<tr>
<td>K</td>
<td>3,500-5,000 K</td>
<td>Red-Orange</td>
<td>Lines of neutral (un-ionized) elements strongest. Ca+ present but weaker. Some molecular absorption features.</td>
</tr>
<tr>
<td>M</td>
<td>2,000-3,500 K</td>
<td>Red</td>
<td>Balmer lines weak. Spectrum dominated by molecular features, such as TiO, C2 and CH.</td>
</tr>
</tbody>
</table>
Spectral Types (4)

- Mnemonics:
  - “Oh, be a fine girl (guy), kiss me (right now, smack)!”
  - “Oh, brutal and fierce gorilla, kill my roommate next Saturday!”
  - “Oven baked ants, fried gently, kept moist, retain natural succulence.”
  - “Our best aid for gnat killing—My reindeer’s nose. Snort.”
  - “Orders by a fat general kill many really nice soldiers.”

- Note presence of three rare categories, RNS, which we will proceed to ignore.
Why Spectra Vary in Appearance

• The original explanation for the variation in the elements that we see in stellar spectra was that there really was a difference in the abundance of the elements in different stars.
  • That is, it was believed that some stars had more hydrogen, some more calcium, etc.
• Cecelia Payne (1920s) showed that this is not true. There are slight abundance variations from star to star (in the elements heavier than helium), but she showed that stars consist of
  • Approximately 90% hydrogen
  • Approximately 10% helium
  • Approximately 1% of everything else (much less than this in some stars).
Variation of Spectrum With Temperature

**FIG. 14-3** Relative intensities of different absorption lines in stars at various places in the spectral sequence.
Variation of Hydrogen Lines With Temperature

Hydrogen Balmer lines

\[ \lambda = 4101 \text{ Å} \quad \lambda = 4340 \text{ Å} \]

(VEGA)

(SUN)

\[ \lambda = 4101 \text{ Å} \quad \lambda = 4340 \text{ Å} \]

9600°K
8800°K
8200°K
7500°K
7200°K
6800°K
5800°K
The Cosmic Abundance

- The strength of a line in a star’s spectrum does not directly tell us how much of that element exists in the star.
  - It depends on the state of ionization of the element.
  - It depends on whether atoms of that element are in the proper energy state to absorb light of that wavelength.
  - It depends on how easy it is for the transition to take place. Ca\(^+\) lines and H Balmer lines are about equally strong in an F2 star, although there is much more hydrogen than calcium.
  - Some substances have no strong lines in the visible spectrum. Their strongest lines may be ultraviolet (e.g., C) or infrared or microwave (e.g., H\(_2\)).
The Cosmic Abundance (4)
The H-R Diagram
Plotting the H-R Diagram (5)

- Here’s another plot of the H-R Diagram. Filled circles are the stars that are brightest in the night sky. Open circles are the stars that are closest to the Sun.
Interpreting the H-R Diagram

• The H-R diagram shows that the brightest stars in the sky are (as a group) very different from the stars closest to the Sun.

• The brighter stars are clearly *unusual*. The reason we see them is that they are unusually bright and can be seen unusually far away. It is like looking at a newspaper. Most of the people in the paper are unusual, that's why there is a story about them. If you walk down the street, most of the people you pass are ordinary. It's the same with stars.

• This is a *selection effect*
The stars closest to the Sun represent the vast majority of all stars.
Interpreting the H-R Diagram (2)

• There are 48 stars in the plot of the nearby stars and 22 stars in the plot of the brightest stars. Of these (excluding the Sun), only Sirius and \( \alpha \) Centauri appear on both plots (but \( \alpha \) Centauri is double, so there are actually 3 points plotted).

• The nearby stars are a sample typical of all stars. Only two of them (other than the Sun) are more luminous than the Sun. That is about 8-10%.

• Hence we can estimate that of all stars, maybe 8-10% are more luminous than the Sun.

• Inferring the populations of various kinds of objects is an important statistical problem in astronomy.
Plotting the H-R Diagram (5)

- The H-R diagram again. Note the stars in the lower left of the diagram.
Interpreting the H-R Diagram (2)

- There are 4 stars in the plot of nearby stars that form a separate group on the lower left. These stars are known as white dwarf stars. They are hot and small (hence the name).
- Again, if the nearby stars are typical, and 4 out of 48 of them are white dwarfs, something around 8-10% of all stars are white dwarfs.
Stellar Radii, Etc.

• Now let’s look at the radii of the stars in the H-R diagram.
  • The amount of energy radiated per second by a star (its luminosity) is related to two things: the amount of energy radiated per second per square centimeter of stellar surface, and the total surface area of the star.
  • That is, Luminosity = (energy radiated per second per unit of surface area) \times (surface area)

• There are two ways for a star to have a large luminosity:
  • Have a large surface area, although the amount of energy radiated per unit area may be smaller than other stars.
  • Radiate a large amount of energy per second, although the total surface area may be smaller than other stars.
Stellar Radii, Etc. (2)

- The diagram shows lines of equal radius for stars, taking into account both the luminosity and the temperature (which controls the luminosity per square area of star surface).
Now we see why we call stars in the lower left “white dwarfs”—they are white and small. We can also see the reason for the names of the other groups. 90% of all stars lie on the Main Sequence.
Explanation of the Main Sequence

- If we plot the masses of main sequence stars against their luminosities, we find yet another relationship. This is the mass-luminosity relationship.
Explanation of the Main Sequence (2)

- There is a relationship between the masses of stars on the main sequence and their luminosities. If we mark this on the H-R diagram, we see a very interesting relationship:
Explanation of the Main Sequence (3)

- This plot shows us that *the main sequence is a mass sequence*. The more massive a main sequence star is, the brighter and hotter it is.

- To understand this, we will consider the physics that goes on inside of stars. For the moment, we will just take it as a fact that we have determined observationally.

- But we can now ask other questions: How much more massive than the Sun are the most massive stars? How much less massive than the Sun are the least massive stars? What is the mass ratio between the most and least massive stars?

- Also, which stars in the universe are most common, those of high mass or those of low mass?

- The distribution of the initial masses of stars is an important statistical problem (IMF - Initial Mass Function)
Stellar Equilibrium

• In stars, the high temperatures vaporize all the material into gas.

• The nuclei can be packed very closely together because the gas is ionized—it is what we call a plasma.

• Why does a star not collapse under its own gravity? What holds a star up?
  • *Gas pressure* tends to counteract the force of gravity. This is the same reason why the Earth’s atmosphere doesn’t collapse down to the surface of the Earth.
  • Also, enormous amounts of radiation stream out from the center of a star, and tend to push matter outwards. This is *radiation pressure*. 
Once a star has arrived on the main sequence it is always in equilibrium. It remains in this state throughout most of its life. At any point in the star, from the inside to the outermost atmosphere, the gravitational force is precisely balanced by radiation pressure. This equality of forces insures the stability of the star.
Stellar Equilibrium (2)

- Stars of low to moderate mass, including the Sun, are supported primarily by gas pressure.

- Stars of high mass are supported primarily by radiation pressure.

- Stars of very high mass, larger than about 70 times the Sun’s mass, probably cannot exist for long. The radiation pressure would be so intense that it would blow away the outer layer of the star.
Stellar Equilibrium (3)

- Whether by gas or radiation pressure, the reason that blobs of gas in the Sun do not rise or fall is because the gravitational forces pulling it down are just balanced by the pressure forces holding it up.
- It is a state of equilibrium.
Other equilibrium situations hold in a star. For example:

- The rate at which energy leaves the surface of the star must equal the rate at which energy is generated in the star’s interior by fusion of hydrogen (main sequence stars) or heavier elements; otherwise energy would either build up in the star (heating the interior) or become depleted (cooling the interior).

- The rate at which energy enters each layer in the star must equal the rate at which it leaves, otherwise the layer would heat up or cool down.

- If gas rises in one part of a star, an equal amount of gas must fall in another part of the star, otherwise the star would get larger or smaller.
Stellar Equilibrium (5)

• Suppose a star generated more energy at the center than could leave through the surface.
  • The interior of the star would get hotter.
  • This would increase the gas pressure in the star.
  • This would tend to make the star get larger.
  • This would decrease the density and pressure, cooling the star off.
• In this way any minor maladjustments will quickly be compensated, and the star will return to its state of equilibrium.
Stellar Equilibrium (6)

- We can now understand the mass-luminosity relation as well.

- Suppose we add mass to a star.
  - What would happen to the gravitational force on the gas?
  - What would happen to the pressure and temperature at the center of the star?
  - When pressure and temperature change in this way, what will happen to the rate of energy generation in the star?
Stellar Equilibrium (7)

• Here are the results of a computer calculation for the Sun:

```
<table>
<thead>
<tr>
<th>Temperature (millions of K)</th>
<th>Distance from center of Sun (thousands of km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>300</td>
<td>400</td>
</tr>
<tr>
<td>500</td>
<td>600</td>
</tr>
<tr>
<td>700</td>
<td>800</td>
</tr>
</tbody>
</table>
```

Temperature profile inside the Sun

Distance from center of Sun (thousands of km)
Stellar Equilibrium (8)

Density profile inside the Sun

Distance from center of Sun (thousands of km)

Density (grams per cc)
The Lifetimes of Stars

- We just saw that the more massive a star is, the more luminous it is (the mass-luminosity relationship). The *main sequence lifetime* of a star is the length of time it stays on the main sequence, until it runs out of hydrogen in its core.

- The main sequence lifetime of the Sun will be about 9 billion years, according to calculations.

  - Consider a very massive star, say 40 times as massive as the Sun.
  
  - Its luminosity is about $10^4$ times that of the Sun, according to the mass-luminosity relationship.
  
  - Roughly how long can we expect such a star to remain on the main sequence?
The Lifetimes of Stars (2)

- Our rough calculation was very crude. The best computer models predict the lifetimes shown in the table:

<table>
<thead>
<tr>
<th>Mass (solar masses)</th>
<th>Lifetime (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>2 million</td>
</tr>
<tr>
<td>30</td>
<td>5 million</td>
</tr>
<tr>
<td>10</td>
<td>25 million</td>
</tr>
<tr>
<td>3</td>
<td>350 million</td>
</tr>
<tr>
<td>1.5</td>
<td>1.6 billion</td>
</tr>
<tr>
<td>1</td>
<td>9 billion</td>
</tr>
<tr>
<td>0.1</td>
<td>thousands of billions</td>
</tr>
</tbody>
</table>
Spectroscopic Parallaxes (Distances)

- If we know the temperature of a main sequence star, we can estimate its absolute luminosity. Then a measurement of its apparent luminosity gives us the distance to the star

![Graph showing the relationship between surface temperature and solar luminosities. The graph includes a scale for surface temperature (Kelvin) and a scale for solar luminosities, with data points scattered across the plot. The Sun is marked on the graph.](image)
**Binary Stars**

- Many stars that appear single to the eye are actually multiple star systems. That is, there are two (or sometimes more) stars, bound together by gravity, orbiting each other in elliptical orbits.

- *Visual binaries*, which can be seen in the telescope as two separate stars, are especially useful to astronomers. Over time, we can watch the two stars orbit around each other. We can measure the size of the orbit, and can calculate its actual size (in A.U.) if we know the distance of the binary. We can also measure the length of time that it takes to complete one orbit—the period of the orbit.
Binary Stars (4)

- Another type of binary is the *spectroscopic binary*. These stars are discovered from evidence in the *spectrum* of a star that indicates that the star is binary.

- Sometimes we see two distinct spectra (usually of differing spectral types), and the spectra shift back and forth over the course of a few days. This shifting is interpreted as being due to the changing redshifts and blueshifts of the two components. Each star takes its turn in moving towards us or away from us.

- Sometimes we see the spectrum of only one star, but the spectrum shifts back and forth in wavelength over the course of a few days. In this case, we conclude that there is a second star, but that the star is too faint for us to see its spectrum in the glare of the other star.
A third type of binary star is the *eclipsing binary*. Here, the orbit is almost edge-on, and periodically one of the stars passes in front of the other one, eclipsing it. We see the star get fainter and then brighter again. The first star of this kind to be discovered is *Algol*, in the constellation Perseus.
Once we know the period, $P$, and the distance between the two stars, $D$, we can use Newton’s modification of Kepler’s third law to estimate the total mass of the system:

$$\text{Total Mass} = \frac{(\text{Distance between the stars})^3}{(\text{Period of the orbit})^2}$$

Data on binary stars is the only direct observational information that we have about the masses of stars.
Statistical Issues

• Observations are often incomplete due to limiting magnitudes of surveys or other causes that are correlated to the thing being measured; this leads to important selection biases (such as the “Malmquist bias”, important for galaxies as well as stars). Methods for accounting for these biases are crude and could be improved.

• Other known biases that need to be, but often are not taken into account, e.g., Lutz-Kelker bias (known to Trumpler much earlier but often ignored).
More generally, one is nearly always limited by the difficulties of sampling astronomical events. Although we may attempt to create catalogs complete relative to some limiting criterion, this is very difficult.

This is exacerbated by incompleteness due to earth rotation, clouds, uneven distribution of telescopes north-south and also in longitude since mountains with good weather are the preferred location for large telescopes.

We often acquire data in a willy-nilly way. A supernova goes off, a telescope just happens to be pointed in the right direction and a minor planet happens to move through. Can sampling theory help us?
Statistical Issues

- Accurate estimation of the IMF is an important research goal; much else depends on this, both theoretically and observationally.
- Information on stellar masses is limited. The observations are difficult and take a long time in many cases to get a significant fraction of the orbit. Also, many methods do not give masses, but are confounded by the orientation of the orbit.
Statistical Issues

• There is always room for improving the comparison between data and theoretical calculations. Often this is done just by eyeball without formal statistical calculation. Since there may be several theoretical models, model selection is inevitably of interest. Astronomers are generally not well trained in statistical methods and may use inappropriate or inadequate methods when they do try to apply them.

• Because of the complex relationship between the strength of spectral lines and conditions in a stellar atmosphere (temperature, pressure), analysis of elemental abundances in stellar atmospheres is difficult and involved. There is doubtless room for improvement by the involvement of professional statisticians.