The Log-Density as a Superior Cosmological Density Variable
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A logarithmic transform makes the low-redshift matter density field better-suited for analysis by the power spectrum. It suppresses sharp peaks in the dark-matter density field, which are conspiracies of Fourier phases, not amplitudes.1

The conventional (overdensity $\delta = \rho / \rho_{\text{mean}} - 1$) power spectrum $P_\delta$ annoyingly departs from a linear shape on small, nonlinear scales. The log transform, $\delta \rightarrow A = \ln(1+\delta)$, largely removes this annoyance.1

The power spectra of the log-density ($P_{\ln(1+\delta)}$) and the rank-order-Gaussianized density ($P_{G(0)}$) have dramatically smaller covariance than $P_\delta$, too.1,4

The reduction in covariance gives $P_{A \ln(1+\delta)}$ a factor of 2-3 tighter constraints on cosmological parameters, e.g. $\sigma_8$. For the tilt $n_s$, the improved fidelity to the linear power spectrum shape enhances constraints, even further, up to a factor of 5. Log-power spectrum emulator available at http://skysrv.pha.jhu.edu/~neyrinck/CosmicEmuLog5

$P_A$ is analytically tractable in renormalized perturbation theory, despite challenges from the nontrivial effects of smoothing and the slowly converging log power series. Even analytically, there are hints that its shape is closer to linear theory than $P_\delta$.3

Using the log-density also substantially improves estimates of the Lagrangian displacement field from the density field (using $A = \nabla \cdot \psi$ instead of $\delta = \nabla \cdot \psi$).6 This should allow smaller scales in the initial conditions to be reconstructed, e.g., for BAO detection.

Discreteness (shot-noise) effects for galaxies are larger than for $P_\delta$, but at a reasonable sampling level, a Gaussianizing transform like the logarithm still reduces nonlinearities in the power spectrum Fisher information and shape.2

For Details:
6Falck, Neyrinck, Aragon-Calvo, Lavaux & Szalay 2011, in prep.