Bayesian Inference for the White Dwarf Initial-Final Mass Relation

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Summary

• Stars lose mass as they age, and understanding mass loss is important for understanding stellar evolution.
• The initial-final mass relation (IFMR) is the relationship between a white dwarf’s initial mass on the main sequence and its final mass.
• We have developed a new method for fitting the IFMR based on a Bayesian analysis of photometric observations, combining deterministic models of stellar evolution in an internally coherent way. No mass data are used.
• Our method yields precise inferences with uncertainties for a parameterized linear IFMR. Our method can also return posterior distributions of white dwarf initial and final masses.

Background: Color-Magnitude Diagrams

• Observe stars’ luminosities through different filters
• For the star clusters we study, several parameters are common to all stars:
  - Chemical composition (metallicity)
  - Age
  - Distance
  - Absorption
• Initial masses vary star to star
• Color-magnitude diagrams show the temperature (horizontal axis) and brightness (vertical axis) of stars in different evolutionary states
• For single-age clusters, these different evolutionary states are determined by stars’ initial masses

Fitting the IFMR

Cluster Star Likelihood

• Gaussian errors:
  \[ Y_i \sim N(\mu_i, \Sigma) \]
• \[ Y_i \] = vector of observations of magnitudes through different filters
• \[ \mu_i = (M_1, M_2) \] = the primary and secondary mass of star \( i \)
• \( \mu_i \) = vector of cluster parameters, including age, metallicity, distance, and absorption
• Observational uncertainties \( \Sigma \) are assumed known
• \( M_1 \) and \( M_2 \) are functions of unknown parameters and depend on deterministic stellar evolution models \( G_{\alpha} \) and \( \alpha \)

Mixture Model for Field Stars

• Field stars appear in observational field of view, but are not part of the cluster.
• For simplicity, field stars are assumed uniformly distributed in magnitude space.
• Mixture model:
  \[ Z_i \sim \text{Bernoulli}(\pi_i) \]
• Field model:
  \[ Y_i \mid M_1, M_2, Z_i \sim p(Y_i \mid M_1, M_2, \theta, \alpha) \]

Prior Distributions

• Primary mass:
  \[ \log_{10}(M_1) \sim N(-1.02, 0.677^2) \]
  0.1 M_1 < M_1 < 8.0 M_\odot

Model Fitting

• Unknown parameters: \( M_1, R, \theta, \alpha \)
• MCMC (Metropolis algorithm) on lower-dimensional marginal distribution \( p(\theta, \alpha \mid Y) \)
• Numerical integration to marginalize over \( (M_1, R) \)
• Because of conditional independence, 2N-dimensional integral factors into N 2-dimensional integrals that can be evaluated in parallel within each MCMC iteration

Results: Hyades

• Analyzed Hyades data after adjusting for different distances to individual cluster members.
• Inferences agree with IFMRs from the literature, without using white dwarf mass data.
• Bimodality due to two possible age solutions, at approximately 525 Myr and 665 Myr.

References