Commentary to *Sparse Astronomical Data Analysis*, by Jean-Luc Starck

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Source detection and Poisson denoising

Evaluation of significance of wavelet coefficients under

- **Gaussian noise**
- **Poisson noise - Anscombe transform**
- **Gaussian + Poisson noise - generalized variance stabilization**
- **Poisson noise with few events - MS-VST**

See for instance Zhang et al. (2008), Dupe' et al. (2009), Starck et al. (2009), Schmitt et al. (2010)

Joint significance of multiple coefficients

Multiple testing techniques (e.g. to identify jointly several sources).

- **Weak lensing mass reconstruction with FDR (Starck et al. (2006))**
- **HC for non-Gaussianity (Jin et al. (2005))**
- **Block thresholding (Chesneau et al. (2008))**

See also Miller et al. (2001), Genovese et al. (2002), Perone Pacifico et al. (2004a,b). Many further statistical developments (see talks on Friday).

Statistical and astronomical challenges

Most (by no means all) of the relevant statistical theory has been developed under the assumption of independent tests (e.g., independent wavelet coefficients).

For astrophysical data, this is almost never the case.

A major issue is then, for instance, to understand the joint dependence structure of wavelet coefficients.

What can the stochastic literature offer?

Gaussian fluctuations in Poisson space

Peccati and Zheng (2011) use Malliavin calculus and Stein's method to provide exact bounds on the joint distribution of multiple functionals of Poisson processes. Their results can be applied to arrays of spherical wavelet coefficients; let

$$W_j = (w_{j1}, ...w_{jn}) \text{ , } F_j \sim N(0, \Gamma_{jF})$$

where $EW_j = 0$, $EW_j W_j^T = \Gamma_{jW}$; then

$$d(W_j, F_j) \leq const \times \|\Gamma_{jW} - \Gamma_{jF}\| + K \sum_{k=1}^{n} \int_{S^2} |\psi_{jk}(x)|^3 dx$$

See also Durastanti et al. (in progress)

Some related issues

Some challenges for statisticians:

- **Characterization of coefficient correlation for Poisson fields**
- **Optimal choice of resolution**
- **Optimal choice of wavelets**
- **Applications to multiple testing**

Further developments

The stochastic literature has witnessed many possible developments of interest:

- **Euler characteristics and Gaussian kinematic formula (Adler and Taylor 2007, Adler et al. 2010)**
- **Multiple peak detection - unknown number (Schwartzman et al. 2010)**
- **Density confidence intervals by empirical process and concentration inequalities (Kerkyacharian, Nickl and Picard 2011, Kueh 2011)**
- **Applications to multiple testing**

Weak lensing and polarization

(See e.g. Starck, Moudden and Bobin (2009)) - some related papers on tensor/matrix estimation:

- **MLE for eigenvalues and eigenvectors - tensor valued fields (Schwartzman et al. 2009)**
- **Soft thresholding and sharp adaptation on vector bundles (Kim 2009)**
- **Matrix deconvolution (Kim 2010)**
- **Manifold estimation (Genovese et al. 2010)**

Weak lensing and polarization

Some related mathematical developments:

- **Relationship with group representation theory (Leonenko and Sakhno 2009, Malyarenko 2009)**
- **Characterization of Geometric features - nodal lines and Minkowski functionals (Wigman 2010, Zelditch 2007-2010)**
- **Relationships with orthogonal polynomials (Jacobi) (Petrushev and Xu 2006-2010)**
- **More abstract geometric structures (Geller and Pesenson 2011)**

Inverse Problems and Deconvolution

(See Dupe' et al. 2009-2011).
A (nearly-optimal) procedure for adaptive spherical deconvolution has been given by Kerkyacharian, Pham Ngoc and Picard (2011). The idea is that one observes

$$Z_i = \varepsilon_i X_i$$

where $Z_i, X_i \in S^2$ and $\varepsilon_i \in SO(3)$, and wishes to estimate f_X.

Deconvolution - KNP 2011

One has

$$f_Z(\omega) = f_\varepsilon * f_X = \int_{SO(3)} f_\varepsilon(g) * f_X(g^{-1}\omega)dg$$

It is then possible to take harmonic transforms, move to wavelet space, and deconvolve by thresholding techniques - the results can be shown to be adaptive over sparse and regular regions, see again Kerkyacharian, Pham Ngoc and Picard (2011).

A natural question is - can this be extended to weak lensing and fiber bundles?

The answer is likely to be yes, using the spin formalism.

The role of Gaussianity

Usually, in statistics the objective is to obtain *large sample* asymptotic results (consistency, asymptotic Gaussianity, efficiency, minimax....) - e.g., one assumes that more and more realizations are available.
In many cosmological applications, there is *a single* observation (CMB temperature, CMB polarization), observed at greater and greater resolution *high-frequency* or *infill* asymptotics.
What kind of consistency results can be obtained in non-Gaussian circumstances under high-frequency asymptotics? Some connections with works by Michael Stein.

Bayesian thresholding

Bayesian thresholding rules have been recently proposed to deal sparsity and image reconstruction on the sphere - Scott (2011) considers the empirical Bayes thresholding rule by Johnstone and Silverman, i.e.

$$\beta_k \sim wDE(\beta|0,1) + (1-w)\delta_0$$

where DE is double exponential, w is estimated from the data and δ_0 is Dirac at zero. The coefficients are then estimated by the posterior median (*soft thresholding*). The *horseshoe prior* is instead $\beta_k \sim N(0, \tau\lambda)$, where τ and λ are positive Cauchy (fat tails) - computationally efficient and close to model averaging.

Bayesian statistics

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years	prior	B'sian	Bayes	tot	prior/tot	B'sian/tot	B's/tot
86-90	46	32	40	563	8.17%	5.68%	7.10%
91-95	46	33	25	598	7.69%	5.51%	4.18%
96-00	38	34	26	533	7.12%	6.37%	4.87%
01-05	34	33	20	411	8.27%	8.22%	4.86%
06-10	42	45	20	601	6.98%	7.48%	3.33%