Transient Detection in Low Resolution, Noisy Images

Darren Homrighausen
Department of Statistics, Carnegie Mellon University

Overview:

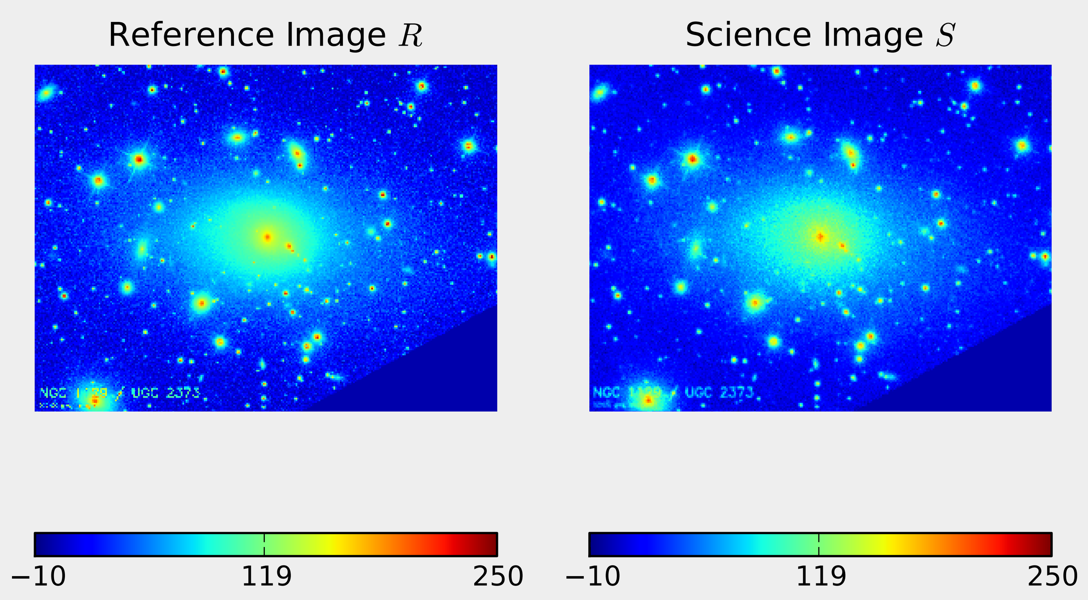

Figure: Left: Reference image R. Result of creating template of scene by combining repeated exposures (coaddition). Right: Science Image S. A new, registered image of the same scene as R, generally lower resolution and noisier.

Existing Methodology:

1. Search and Estimate. For $i = 1, \dots, n$ find a local convolution operator $\hat{k}_i$ such that

$$\hat{k}_i = \text{argmin}_k f\left(\int k(\cdot - y)R_i(y)dy, S_i(\cdot)\right)$$

where f is a discrepancy function such as penalized least squares. See Figure for definition of R_i and S_i.

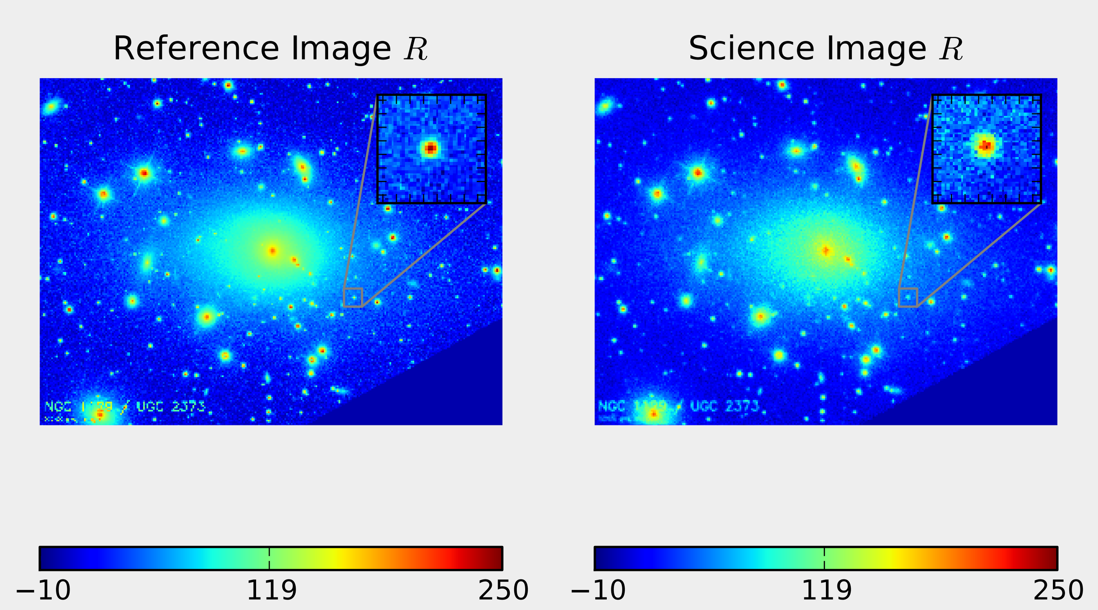

Figure: Left inset: i^{th} reference subimage R_i. Right inset: i^{th} science subimage S_i. Goal of 'Search and Estimate' is to find functions $\hat{k}_i$ that map each R_i to each S_i.

See e.g. Alard, Lupton (1998), Becker, Homrighausen, et al. (2010).

2. Interpolate. Find κ by estimating spatial model for coefficients of $(\hat{k}_i)_{i=1}^n$ in some basis (PCA).
3. Integrate: Compute $\tilde{S}$ as in (1).
4. Detect: Form $\Delta = S - \tilde{S}$ and use SExtractor.

Limitations:

- Each step is considered independently
  - No "sharing of information"
- No accounting for uncertainty

Scientific Goal and Problems

Goal:

- Detect changes between R and S.

Problems:

- R is generally much higher quality.
- Strong computational constraint (near real time processesing).

Our Approach:

1. View κ as a Gaussian process on $\mathbb{R}^2 \times \mathbb{R}^2$ *not* constrained to be a convolution and interpolate between $(\hat{k}_i)_{i=1}^n$:

$$\kappa \sim GP(\mu_\kappa, C_\kappa)$$

where

- $\mu_\kappa(x, y) \equiv \mu_\kappa(x - y) = \sum_{j=1}^p \beta_j \psi_j(x - y)$
  - Endow with prior $\beta \sim N(0, \Sigma_\beta)$
- $C_\kappa((x, y), (x', y')) = \rho_\kappa(x, x', y, y') + \tau^2 \delta_{(x,y),(x',y')}$
- $\rho_\kappa = \rho_\lambda \rho_\gamma$ or $\rho_\kappa = \rho_\lambda + \rho_\gamma$ are covariance functions on $\mathbb{R}^2 \times \mathbb{R}^2$ e.g.

$$\rho_\lambda(x, y) = \sigma_1^2 \exp\left\{-\frac{(x - y)^2}{2\lambda}\right\}, \; \rho_\gamma(x, y) = \sigma_2^2 \exp\left\{-\frac{(x - y)^2}{2\gamma}\right\}.$$

2. This induces a Gaussian Process on $\mathbb{R}^2$ via:

$$\tilde{S}(x) = \int \kappa(x, y)R(y)dy \quad (1)$$

which implies

$$\tilde{S} \sim GP(0, C) \quad (2)$$

where:

- $C(x, x') = \rho(x, x') + \mu(x, x') + \tau^2 \delta_{x,x'}$.
- $\rho(x, x') = \int \rho_\kappa(x, x', y, y')R(y)R(y')dydy'$.
- $\mu(x, x') = \boldsymbol{\varphi}(x)^\top \Sigma_\beta \boldsymbol{\varphi}(x')$ where

$$\boldsymbol{\varphi}_j(x) = \int \psi_j(x - y)R(y)dy.$$

  - convolutional $\Rightarrow O(n \log n)$ computations.

3. Form $\Delta = S - \tilde{S}$.

4. Detect using Δ and a combination of Friedenberg, Genovese (2009) and/or SExtractor.
   - Note that prior model on κ gives principled null hypothesis (conditional on S)

Simulations:

Images:

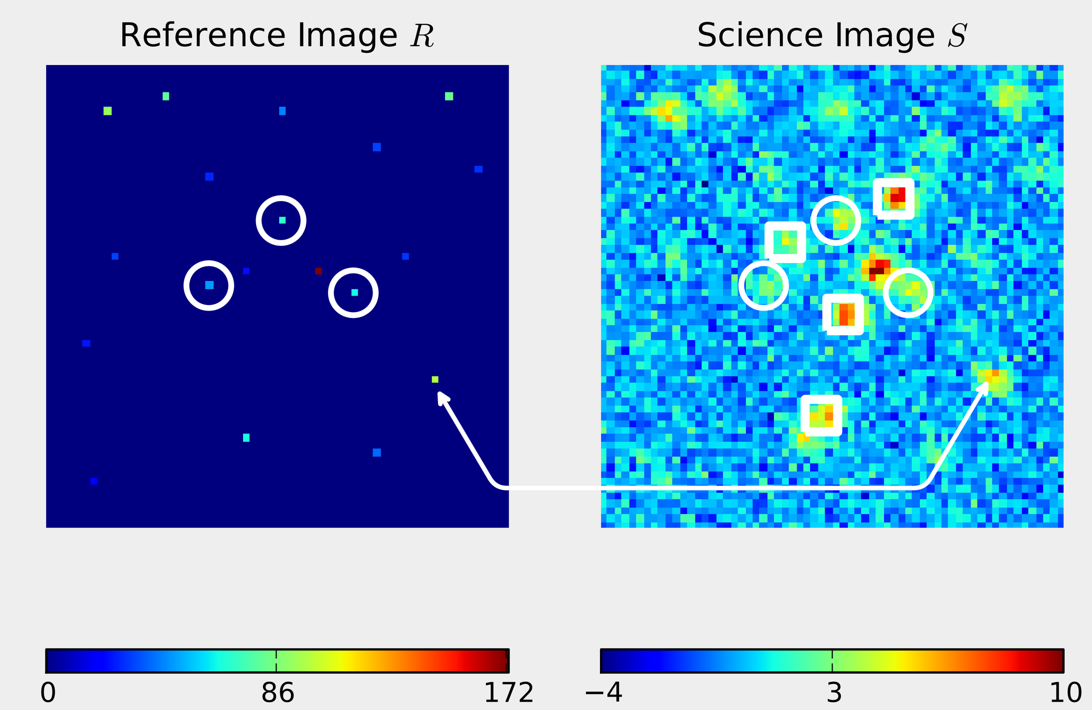

Figure: Left: Reference image R. Circles are objects that *aren't* trained on. Right: Science image S. Each object in R corresponds to a smoothed, noisy object in S (white arrow is an example of one such pairing).

Results:

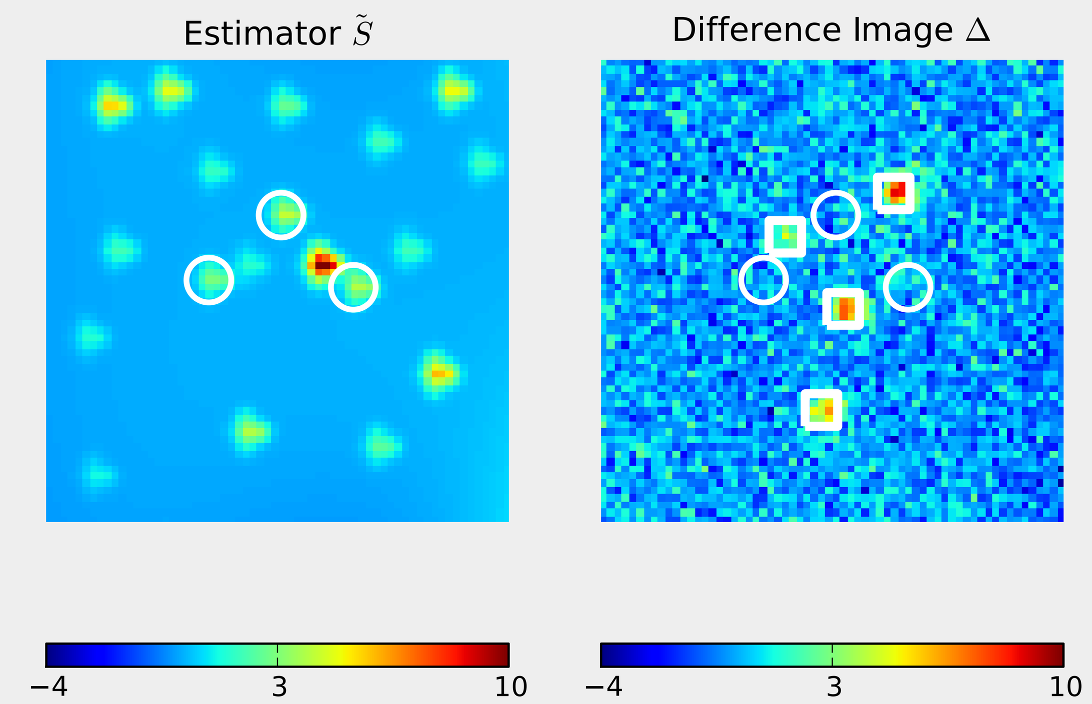

Figure: Left: Output of estimator, $\tilde{S}$. Circles are objects in reference image that *aren't* trained on. Right: Difference image Δ. The non-training points (circles) have been eliminated and the transients (squares) remain.

Discussion:

- This approach provides a very flexible platform for combining the interpolation and integration steps.
- Techniques for fitting the model are plentiful:
  - Fully Bayesian (specify hyperparameters e.g. Σ_β),
  - Empirical Bayes (estimate hyperparameters and some parameters e.g. λ),
  - Fully maximum likelihood (estimate all hyperparameters and parameters via the Gaussian likelihood).

http://www.stat.cmu.edu/~dhomrigh
dhomrigh@stat.cmu.edu