Modeling Undetectable Flares

Vinay L. Kashyap[1], Steve Saar[1], Jeremy Drake[1], Kathy Reeves[1], Jennifer Posson-Brown[1], Alanna Connors[2]

[1]Smithsonian Astrophysical Observatory, [2]Eureka Scientific [vkashyap@cfa.harvard.edu]

SUMMARY

A fundamental characteristic of solar and stellar flares is that the processes that generate them appear to be scale-free. That is, the distribution of flare energies are power-laws. While there is evidence that the power-law model is invalid at very high and very low energies, this distribution has been verified to hold on the Sun over many orders of magnitude of flare energies and over the range of timescales that are accesible to current high-energy astronomy missions (Aschwanden et al. 2000, ApJ, 535, 1047). The number of flares at any given energy range, $(E, E+dE]$, follows a power-law distribution,

$$dN \propto E^{-\alpha}\, dE\,, \qquad (1)$$

where $\alpha \approx 1.8$ for the Sun.

Similar behavior is suspected on active stars, but some crucial differences exist. The first is that due to sensitivity limitations, we cannot explore the behavior of stellar flares at energies below the so-called milliflare region ($E\sim10^{29-32}$ erg). Second, the value of α is generally greater than 2 (see, e.g., Kashyap et al. 2002, ApJ, 580, 1118). The threshold $\alpha=2$ is critical because beyond that, it is possible to ascribe all of the coronal luminosity to increasingly weaker, but more numerous, flares.

It has thus become necessary to systematically study the flare distributions on stars. Unfortunately, current methods to evaluate the flare distribution index α for stars are limited by two factors: they either depend on explicit detections of flares (which limits the analysis to strong flares), or if the flare distribution itself is being modeled, then they are highly computation intensive and are thus slow.

We first developed a method to model the X-ray data directly without resorting to detecting the flares in the first place, by sampling flare energies from an assumed distribution, constructing photon arrival time data stochastically, and comparing the simulated distributions of arrival time differences with that seen in the data (Kashyap et al. 2002). Initial applications of this method were extremely slow because the model distribution of arrival time differences δt had to be empirically generated and the parameter fitting was carried out on a grid.

Here we have speeded up the process considerably by (a) switching to a Markov Chain Monte Carlo fitting method, and (b) computing the model semi-analytically. We demonstrate the new method and show that it gives the same result as before by computing the flare distribution model parameters for the dM3.5 flaring star Ross 154 (see Wargelin et al., 2008, ApJ, 676, 610).

The Problem of Small Flares

It may seem counterintuitive to discuss the contribution of small flares when they are not detected, but because we assume a specific functional form for their distribution (as is justified by observations), their cumulative effect can be easily discerned in the data. Because flare onset is stochastic, it is not feasible to model every feature in a light curve, but rather, they must be modeled only in the aggregate. This intrinsic stochasticity also requires that a large number of simulations be carried out to avoid biasing the result due to a fluctuation, and the overall process has a high computational cost.

We have speeded up the process by replacing the empirically determined distribution of the arrival time differences by an analytical calculation. Recognizing that for a given counting rate R, the probability of finding exactly one event in a duration δt is

$$p(1|R,\delta t) = (R\,\delta t)\, e^{-R\,\delta t} \qquad (2)$$

and if $R=R(t_i)$ is varying, the overall distribution is the sum of the distributions in the interval $[t_i, t_i+\tau]$, weighted by the expected number of events,

$$f(\delta t) = \sum_i R_i\,\tau \cdot p(1|R_i,\delta t)\,. \qquad (3)$$

We also achieve a considerable speed increase by discarding the grid-based parameter probability evaluations and using Markov Chain Monte Carlo methods to efficiently explore the parameter space.

Figure 1 (below): Example light curves and their aggregate properties. The distributions of the count rates (middle panels) and the arrival-time differences (bottom panels) are shown for a variety of light curves, ranging from flat (left) to a flare distribution with α=3. Note that for steep power-laws (α>3), the light curves usually cannot be distinguished from a steady rate. The analytically calculated *f(δt)* are shown in the bottom panels as the green curve. The average count rate (red horizontal line in top panels) and the corresponding distribution of count rates (blue Gaussians in middle panels) are also shown.

Parameter Search

Previously, the likelihood was computed on a 3-dimensional grid of the parameters α, $r_{\rm flare}$, and $r_{\rm base}$, where the latter two rates refer to the average contribution that can be attributed to the flare component and a base component. This was inefficient, since simulations had to be carried out at a large number of grid points, even those that did not produce a large likelihood.

Now, we have switched to using an MCMC[M] algorithm, where the change in likelihood at each iteration is calculated via the change in the value of the χ^2 for both the binned-intensity distribution (middle panels of Figure 1) and the distribution of arrival time differences (bottom panels of Figure 1). The independence of the two sources of χ^2 is ensured by limiting the range of δt that is considered. When the events are binned as light curves at a bin size of $\tau_{\rm bin}$, all information at times smaller than that is lost, but is recovered in the arrival time difference distribution $f(\delta t)$; thus, we have $\max(\delta t) \leq \tau_{\rm bin}$.

Using an MCMC approach is also beneficial in that systematic variations that occur due to the stochastic nature of the model function that evaluated at each iteration. Following Lee et al. (2011, ApJ, 731, 126), we construct a new model light curve at each iteration by randomly drawing a set of flares for the given parameter values. Lee et al. drew instrument effective areas from a calibration sample randomly at each iteration, and equivalently, we draw light curves that differ in the number of flares, their intensities, and the number of superpositions. The current set of parameter values are evaluated against this new light curve, and are accepted or rejected according to the standard Metropolis rule.

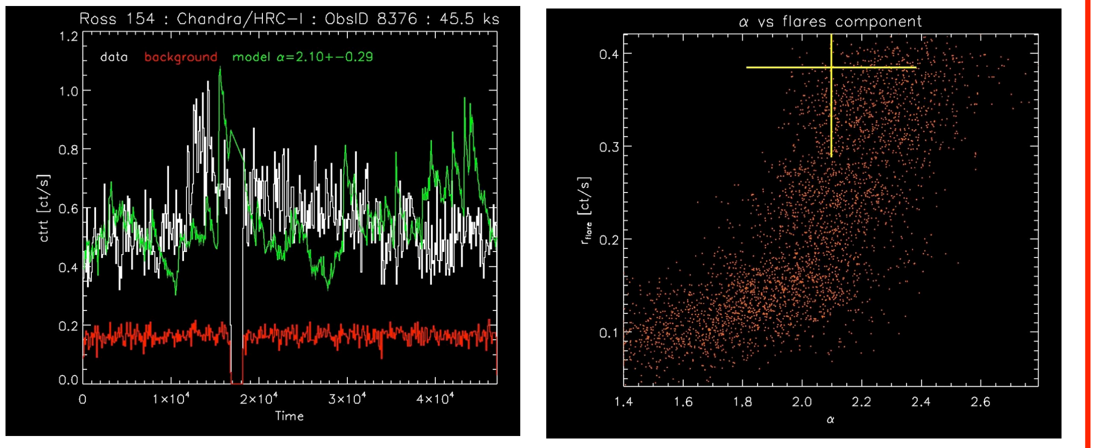


Figure 2 (above): Applying the method to Chandra ACIS-undercover HRC-I data of Ross 154. The observed light curve is shown at left (white histogram), along with the estimated background (red histogram) and the best-fit model (green histogram). Note that the model does not have to match the observed light curve. A scatterplot of the MCMC iterations for the flare index α and the count rate attributable to the flare component, $r_{\rm flare}$ is shown at right, along with the location of the best-fit solution and ±1σ error bars. The most probable value of α is 2.1, but values below 2 cannot be ruled out. The scatterplot shows that two distinct types of solutions are possible -- one with high flare rate and α>2, and another with a low flare rate and α<2. This is an indication that flare distributions on active stars may not be simple power laws. We plan to extend this analysis to a systematic study of a large sample of active stars observed with the Chandra gratings.

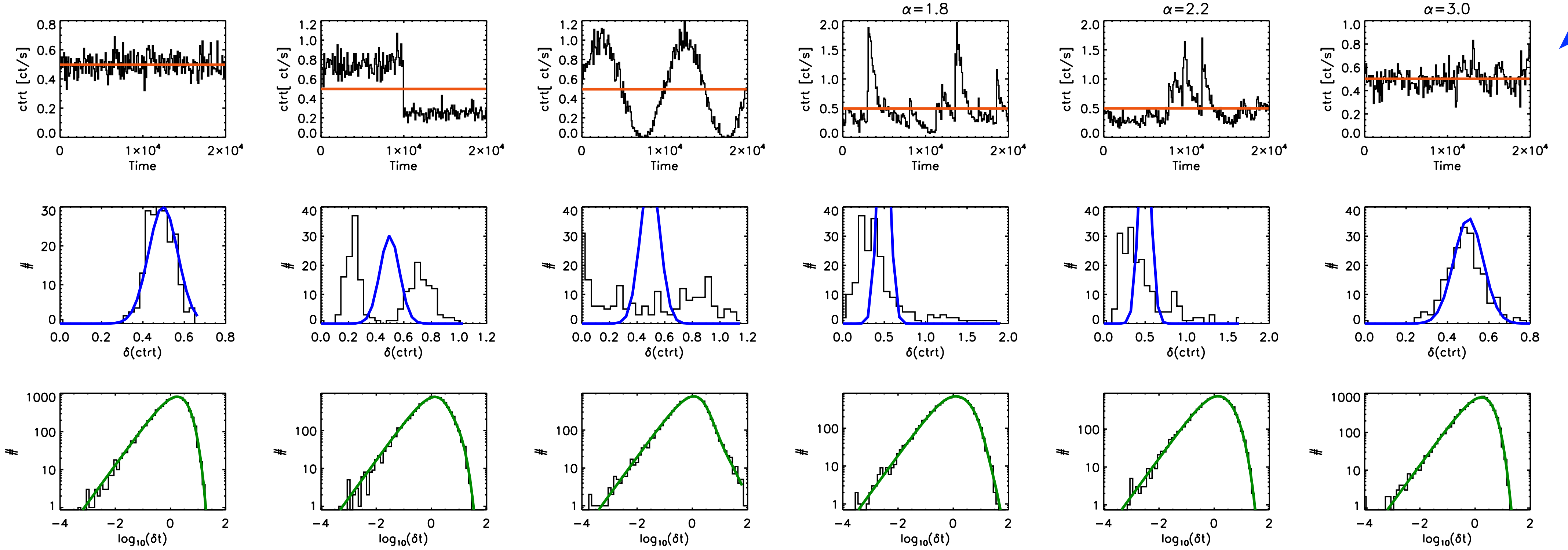
REFERENCES

Aschwanden, M.J., et al., 2000, ApJ, 535, 1047

Kashyap, V.L., Drake, J.J., Güdel, M., & Audard, M., 2002, ApJ, 580, 1118

Lee, H., et al., ApJ, 731, 126

Wargelin, B.W., et al., 2008, ApJ, 676, 610



This work was supported by CXC NASA contract NAS8-39073 and Chandra grant AR0-11001X

SCMA V : Jun 12-17 2011 : State College, PA