A Stochastic Model for the Luminosity Fluctuations of Accreting Black Holes

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Abstract

We present a new statistical model for the X-ray fluctuations of accreting black holes, based on a mixture of independent stochastic processes. Our model is a solution to the linear stochastic diffusion equation, and is consistent with the observed broken power-law form for power spectral densities of these objects. However, because the model is formulated in the time domain via a set of stochastic differential equations, fitting the model is also done in the time domain via a Bayesian approach. This makes it a valuable complement to frequency-domain based methods, in that it provides astrophysical insight; is computationally efficient; and is not biased by red noise leak, aliasing, irregular sampling, or measurement error. We apply our model to combined RXTE + XMM X-ray lightcurves of 10 bright local AGN and show that our model is both a good fit to the data, and is able to recover previous results.

Motivation

- AGN and galactic black holes (GBH) display similar variability properties, important for understanding how physics of the accretion flow scale with mass
- Fluctuations may represent how the accretion flow `responds' to a perturbation, probes viscosity and structure of accretion flow
- Dependence of characteristic time scales on black hole mass and accretion rate provides test of simple accretion flow solutions

The Stochastic Model

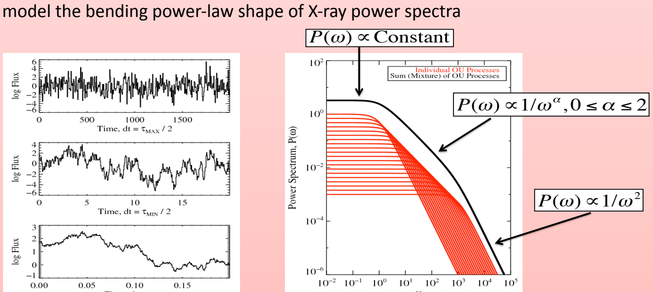
- Kelly et al. (2009) showed that AGN optical variability on time scales of ~ 3 days to ~ 7 years is well described by an Ornstein-Uhlenbeck (OU) process
- Application of the OU process to larger samples (Kozłowski et al. 2010, MacLeod et al. 2010) has confirmed this, and the model has also been applied to sub-mm lightcurves (Strom et al. 2010)
- The OU process, L(t), with mean μ, characteristic time scale $\tau_c = 1 / \omega_c$ and white random driving noise σW(t) is the solution to the stochastic differential equation

$$dL(t) = -\omega_c (L(t) - \mu)dt + \sigma W(t)$$

- The power spectrum of the OU process is a Lorentzian

$$P(\omega) \propto (\omega_c^2 + \omega^2)^{-1}, \;\; P(\omega) \propto 1 \; (\omega << \omega_c), \;\; P(\omega) \propto \omega^{-2} \; (\omega >> \omega_c)$$

- We use a mixture of independent OU processes on a grid of characteristic time scales to model the bending power-law shape of X-ray power spectra



Simulated lightcurves seen on different time scales (left) for the mixed OU process and power spectrum for the mixed OU process (right). The mixed OU stochastic process is a sum of independent OU stochastic processes. The upper and lower breaks In the power spectrum correspond to the maximum and minimum characteristic time scales of the OU processes used in the summation.

- Because the mixed OU process has a State Space representation, we use the Kalman recursions to calculate the likelihood function.
- We fit the model using a Bayesian approach directly on the logarithm of the lightcurve itself, **no Fourier transforms are used**

References

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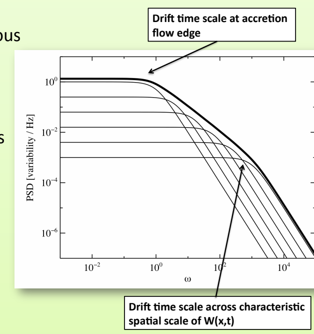
Astrophysical Interpretation

- Consider the linear diffusion equation for a medium subject to a random spatially-correlated driving noise field:

$$\frac{\partial}{\partial t} y(x,t) = a \frac{\partial^2}{\partial x^2} y(x,t) + \frac{\partial}{\partial t} W(x,t)$$
$$y(x,0) = y_0(x)$$
$$y(0,t) = 0 = \frac{\partial}{\partial t} y(x_{\max}, t)$$

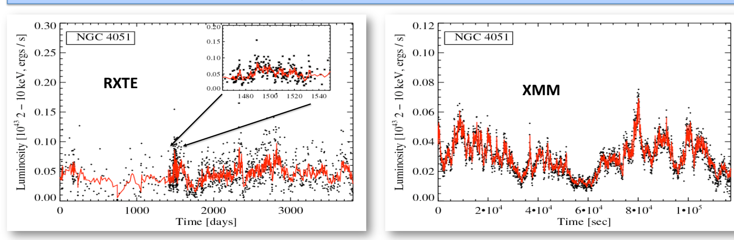
y(x,t): Quantity undergoing diffusion (e.g., mass, heat, etc)
W(x,t): Gaussian noise field, white noise in time but correlated in space
a: Diffusion constant

- The solution, y(x,t), is a mixed OU process at each location x (Chow 2007)
- Under this interpretation, the accretion disk structure acts as a low-pass filter on the driving noise
- The lower break frequency corresponds to the viscous timescale near the outer edge of the region experiencing the random perturbations
- The upper break frequency may correspond to the crossing time for a perturbation traveling across the characteristic spatial scale of the driving noise field

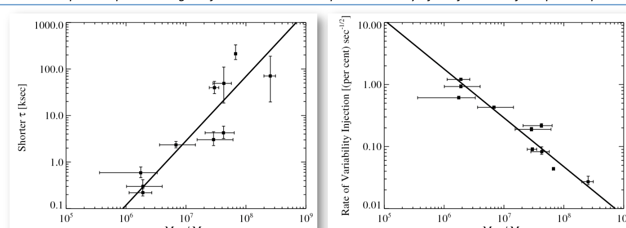


Application to AGN X-ray Lightcurves

- Used our stochastic model to fit the lightcurves of 10 AGN with long, high quality X-ray lightcurves from RXTE (Sobolewska & Papadakis 2009)
- Also included XMM lightcurves, when available
- Probes time scales of ~ seconds to ~ years



Observed lightcurve obtained from RXTE (left) and XMM-Newton (right), compared with the running average predicted from the best-fitting mixed OU stochastic process. The scatter about the running average is due to a combination of the Poisson noise and random fluctuations caused by the driving noise; the running average is the expected value of the stochastic process given the previous values, and will in general not equal the actual realization of the stochastic process. The mixed OU process provides a good fit to the data and reproduces many of the features of the power spectrum.



Dependence on black hole mass of the characteristic time scale corresponding to the high frequency break in the X-ray power spectrum (left) and the rate at which variability power is 'injected' into the lightcurve (right). The amplitude of very short time scale variability is proportional to the rate of variability power injection. Tight correlations are apparent, and confirm earlier trends found by, e.g., McHardy et al. (2006) regarding the characteristic time scale and Zhou et al. (2010) regarding the short time scale variability amplitude.