Statistical differences between Swift GRB classes based on γ- and X-ray observations

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Abstract

There are a number of notable evidences that the Gamma-ray Bursts have a third group beside the short and long ones: the intermediate group; although at this time, no reasonable physical explanation is known for them. We give a new independent analysis based on the statistics of the observational parameters in order to prove the separation between the three groups, particularly between the long and the intermediate groups. Using discriminant analysis of the multivariate mathematical statistics, we analysed the γ- and X-ray properties observed by the Swift satellite. We present the discriminating functions giving the maximum separation between the groups and the statistical parameters dominating the discriminating functions. Our result confirms the former classification based on the hardness-duration joint distribution and give some further physical properties, which could be useful in the future to construct or develop a model for the intermediate GRBs.

Introduction

Discriminant analysis is a useful statistical tool for differentiating between groups of cases and to classify objects with known quantitative parameters. It is based on the demonstrable statement that assigning an object, defined by a set of variables (observations), to a class can be done by constructing a function of a well-chosen linear combination of the known observations. We used this algorithm to confirm the classification of the GRBs based on the hardness-duration joint distribution (see Horvath et al. (2010)). Now we analysed the γ- and X-ray properties observed by the Swift satellite. The burst alert given by the BAT, which is a γ-energy range telescope on board of the satellite, is followed by the XRT detection in the X-ray regime, and a significant fraction of GRBs is observed in both energy ranges. The variables used in this work were the following: *Fluence*, *1-sec Peak Photon Flux*, *Photon Index*, *Early X-Flux*, *Initial Temporal Decay Index*, *Spectral Index* and *HI Column Density* (*XNH*).

Mathematical Summary

For an $\{\mathbf{X}_i\}$ set of observations (containing n cases and p parameters) and k classes, the $\mathbf{y}$ canonical (linear) discriminant function is:

$$\mathbf{y} = a_1\mathbf{x}_1 + a_2\mathbf{x}_2 + \cdots + a_p\mathbf{x}_p. \quad (1)$$

The coefficients a_p are chosen by maximalizing the separation between the groups, assuming $\sum_{i=1}^{p} a_i^2 = 1$. Since we have k groups, we get $k-1$ discriminant functions (if $k-1 < p$). To get the canonical discriminant functions we construct the cross-product matrices for between-group deviations S_b and within-group deviations S_w, then we maximalize the ratio of them. Thus, we get an eigenvalue-equation, which gives us $k-1$ eigenvectors and eigenvalues (λ_i). The elements of the eigenvectors give the discriminant function coefficients a_p.
Wilks' lambda is the ratio of the determinants of the matrices S_w and $S_b + S_w$:

$$\Lambda = \frac{|S_w|}{|S_b + S_w|} = \prod_{i=1}^{k-1} \frac{1}{1+\lambda_i}. \quad (2)$$

The significance of the differences between the groups with the χ^2 probe (degrees of freedom: $p(k-1)$) is:

$$\chi^2 = -\left(n - \frac{p+k}{2} - 1\right) \log \Lambda. \quad (3)$$

Therefore we can calculate the probability that the classes differ only by chance.

Data Preparations

For large set of observations, one can use professional statistical software packages[1]. We used SPSS[2] in our computations.
In our data set, we have 11 short, 61 intermediate and 123 long GRBs based on the grouping of the hardness-duration joint distribution. First, we verified this grouping using discriminant analysis.

Test of Functions	Wilks' Lambda	Chi-square	df	Sig.
1 through 2	.289	234.926	14	.000
2	.745	55.530	6	.000

Table 1: Significance of the differences measured by the discriminant functions.

We can see in Table 1 that the difference between the defined groups is significant at a high level.

[1] e.g. in the R package discriminant analysis is performed by the `lda` procedure.

[2] SPSS is a registered trademark (`http://www.spss.com`).

Discriminant Analysis: Separation Between Long And Intermediate Groups

We used discriminant analysis in order to confirm the separation between the long and the intermediate groups, and to get the discriminant function and the statistical parameters dominating the discriminant function. Means and standard deviations are summarized in Table 2.

Variable	Mean of Iterm.	Std.dev. of Interm.	Mean of Longs	Std.dev. of Longs
Pind	**1.86**	**.49**	**1.50**	**.33**
Xdec	2.07	1.68	2.24	1.53
Xsp	**2.23**	**.75**	**1.95**	**.44**
logFl	**.62**	**.42**	**1.36**	**.48**
logP	.09	.40	.22	.49
logXfl	**.59**	**1.01**	**1.56**	**1.01**
logXNH	**.13**	**.55**	**.35**	**.49**

Table 2: Means and standard deviations of the variables in the analysis. Bold face: mean values differ significantly between the groups, see Table 3.

In Table 3, we compared the means of the variables between the groups using F-statistics. Bold faces mark the variables where the differences in the group means are significant.

Variable	Wilks' Lambda	F	df1	df2	Sig.	Corr.
Pind	**.838**	**35.055**	**1**	**182**	**.000**	**-.409**
Xdec	.997	.476	1	182	.491	.048
Xsp	**.943**	**10.908**	**1**	**182**	**.001**	**-.228**
logFl	**.634**	**104.841**	**1**	**182**	**.000**	**.707**
logP	.982	3.258	1	182	.073	.125
logXfl	**.833**	**36.385**	**1**	**182**	**.000**	**.417**
logXNH	**.960**	**7.660**	**1**	**182**	**.006**	**.191**

Table 3: Test of significance of the differences in the mean values between the groups. Last column: pooled within-groups correlations between discriminating variables and standardized canonical discriminant functions.

In our case, we have two classes. The discriminant analysis calculated one discriminant function to discriminate between the classes. The correlation between the variables and the discriminant function is shown in the last column of Table 3. The correlation coefficients marked with bold faces have the highest absolute values (intriguingly, these are the ones where the differences in the group means are significant based on F-statistics). Therefore, the discriminant function is mostly dominated by the γ-Fluence and less by the Early X-Flux, the Photon Index, the Spectral Index and the HI Column Density.

Test of Function	Wilks' Lambda	Chi-square	df	Sig.
1	.465	136.721	7	**.000**

Table 4: Significance of the difference between long and intermediate groups measured by the discriminant function. Analysing this table, we can pronounce that there is a significant difference between long and intermediate groups.

As the value in the column Significance is **.000**, we can state that the two groups differ significantly based on the γ- and X-ray observations.

Conclusion

Our result indicates that the classification of GRBs based on the hardness-duration distribution has also an influence on the γ- and X-ray properties. Additionally, we confirmed the separation between the long and intermediate groups and gave the variables dominating the discriminant function (Fluence, Early X-Flux, Photon Index, Spectral Index and HI Column Density). It means that one have to take the set of these γ- and X-ray variables into account when constructing or developing a model for the intermediate GRBs.

References

Horvath, I. et al. 2010, ApJ, 713, 552

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